Simulations of qubit communication in prepare-and-measure and Bell scenarios

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OUTLINE:

1. INTRODUCTION
2. OBJECTIVES
3. CLASSICAL SIMULATIONS
4. QUANTUM SIMULATIONS WITH QUANTUM COMPUTERS
5. CONCLUSION AND FURTHER WORK

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1. INTRODUCTION

INTRODUCTION

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M. J. Renner, A. Tavakoli, M. T. Quintino, 2022,
The classical cost of transmitting a qubit

Considering a **general prepare-and-measure scenario** in which Alice can transmit qubit states to Bob, who can perform general measurements in the form of **positive operator-valued measures**…

… the **statistics** obtained in such scenario **can be reproduced by** purely classical means of **shared randomness** and **two bits** of communication.

Furthermore, it is proved that two bits of communication is the minimal cost of a perfect classical simulation.

In addition, the protocol can be adapted to **Bell scenarios**, extending Toner and Bacon results. In particular, **one bit of communication** is enough to **reproduce all quantum correlations** associated to arbitrary local measurements applied to a **Bell singlet state**.

Can we simulate and verify this?

LET´S CHECK…

The classical cost of transmitting a qubit

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(Dated: 7th July 2022)

We consider general prepare-and-measure scenarios in which Alice can transmit qubit states to Bob, who can perform general measurements in the form of positive operator-valued measures (POVMs). We show that the statistics obtained in any such quantum protocol can be simulated by the purely classical means of shared randomness and two bits of communication. Furthermore, we prove that two bits of communication is the minimal cost of a perfect classical simulation. In addition, we apply our methods to Bell scenarios, which extends the well-known Toner and Bacon protocol. In particular, two bits of communication are enough to simulate all quantum correlations associated to arbitrary local POVMs applied to any entangled two-qubit state.

Introduction.— Quantum resources enable a sender and a receiver to break the limitations of classical communication. When entanglement is available, classical [1–4] as well as quantum communication [5, 6] can be boosted beyond purely classical models. A seminal example is dense coding, in which two classical bits can be substituted for a single qubit and shared entanglement [7]. However, entanglement is not necessary for quantum advantages. Communicating an unassisted d-dimensional quantum system frequently outperforms the best conceivable protocols based on a classical d-dimensional system [8–12]; even yielding advantages growing exponentially in d [13, 14]. Already in the simplest meaningful scenario, namely that in which the communication of a bit is substituted for a qubit, sizable advantages are obtained in important tasks like Random Access Coding [15–17]. These qubit advantages propel a variety of quantum information applications [18–22].

It is natural to explore the fundamental limits of quantum over classical advantages. In order to do so, one has to investigate the amount of classical communication required to model the predictions of quantum theory. Previous works consider not only the scenario of sending quantum systems [23–27], but also simulating bipartite [23–33], as well as multipartite entangled quantum systems [34–36]. While such classical simulation of quantum theory is in general challenging, a breakthrough was made by Toner and Bacon [26]. Their protocol shows that any quantum prediction based on standard, projective, measurements on a qubit can be simulated by communicating only two classical bits. However, this does not account for the full power of quantum theory. More precisely, there exists qubit measurements that cannot be reduced to stochastic combinations of projective ones [37]. The most general measurements are known as positive operator-valued measures (POVMs). Physically, they correspond to the receiver interacting the message qubit with a locally prepared auxiliary qubit, and then performing a measurement on the joint system [38]. Such POVMs are even indispensable for important tasks like unambiguous state discrimination [39, 40] and hold a key role in many quantum information protocols (see e.g. [41–49]). Importantly, they also give rise to correlations that cannot be modelled in any qubit experiment based on projective measurements [50–54].

This naturally raises the question of identifying the classical cost of simulating the most general predictions of quantum theory, based on POVMs. In the minimal qubit communication scenario, one may suspect that this cheap price of only two bits is due to the restriction to the, fundamentally binary, projective measurements. In contrast, when measurements are general POVMs, it is even unclear whether the classical simulation cost is finite. Notably, previous work has shown that there exists a classical simulation that requires 5.7 bits of communication on average [23, 27]. However, that protocol has a certain probability to fail in each round, leading to an unbounded amount of communication in the worst case.

In this work, we explicitly construct a classical protocol that simulates all qubit-based correlations in the prepare-and-measure scenario by using only two bits of communication. Thus, we find that the cost of a classical simulation remains the same when considering the most general class of measurements, although POVMs enable more general quantum correlations than projective measurements. Moreover, we show that two bits is the minimal classical simulation cost, i.e. there exists no classical simulation that uses less communication than our protocol. This is shown through an explicit quantum protocol, based on qubit communication, that eludes simulation with a ternary classical message. Finally, we apply our methods to Bell nonlocality scenarios. We present novel protocols that simulate the statistics of local measurements on entangled qubit pairs.

The prepare-and-measure scenario.— A quantum prepare-and-measure (PM) scenario (see Fig. 1 a)) consists of two steps. Firstly, Alice prepares an arbitrary quantum state of dimension d_Q and sends it to Bob. The state is described by a positive semidefinite $d_Q \times d_Q$ complex matrix $\rho \in \mathcal{L}(\mathbb{C}_{d_Q})$, $\rho \geq 0$ with unit trace $\mathrm{tr}(\rho) = 1$.

[26] B. F. Toner and D. Bacon, Communication Cost of Simulating Bell Correlations, Phys. Rev. Lett. **91**, 187904 (2003), arXiv:quant-ph/0304076 [quant-ph].

2. OBJECTIVES

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MAIN OBJECTIVE:

Prove by **computer-based experiments** that a **qubit communication** can be simulated classically with a total cost of **2 classical bits for any POVM** in a **prepare-and-measure scenario**, or **1 classical bit for any arbitrary local measurement in a Bell scenario.**

PROJECT STEPS TO ACHIEVE IT:

✅ **Simulate Prepare-and-Measure PVMs classically**

- ✓ **Generation of random states and projection-valued measurements**
- ✓ **Classical simulation**

✅ **Simulate Prepare-and-Measure with POVMs**

- ✓ **Generation of POVM measurements**
- ✓ **Classical simulation**
- ✓ **Quantum Simulation using IBM Quantum resources**

✅ **Simulate using Bell scenarios**

- ✓ **Bell's singlet and local projective measurements**
- ✓ **CHSH inequality**

https://github.com/inaki-ortizdelandaluce/qubit-communication-simulations

3. CLASSICAL SIMULATIONS

PREPARE-AND-MEASURE SCENARIO

Alice prepares **qubit state ρ** and sends

Bob receives state ρ and **measures** with **POVM $\{B_k\}$**

$$p_Q(k|\rho,\{B_k\}) = tr(\rho B_k)$$

$$p_C(k|\rho,\{B_k\}) = \int_\lambda d\lambda\, \pi(\lambda) \sum_{c=1}^{d_C} p_A(c|\rho,\lambda)\, p_B(k|\{B_k\},c,\lambda)$$

Classical information

Qubit state

Shared randomness

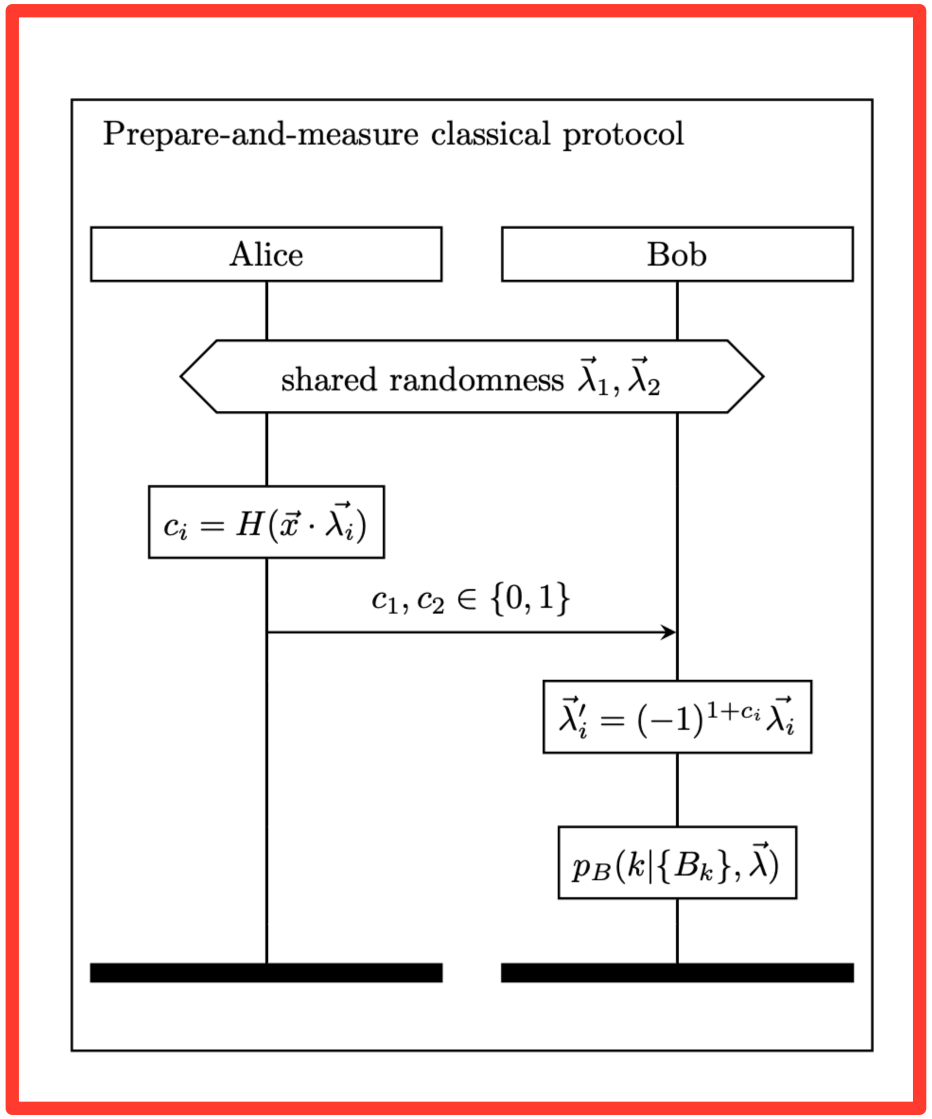
POVM measurement

$$\forall \rho,\{B_k\}: \quad p_C(k|\rho,\{B_k\}) = p_Q(k|\rho,\{B_k\})$$

Success criteria

3. CLASSICAL SIMULATIONS

PREPARE-AND-MEASURE CLASSICAL PROTOCOL



1. Alice and Bob share two normalized vectors $\vec{\lambda}_1, \vec{\lambda}_2 \in \mathbb{R}^3$, which are uniformly and independently distributed on the unit radius sphere S_2.
2. Instead of sending a pure qubit $\rho = (\mathbb{1} + \vec{x} \cdot \vec{\sigma})/2$, Alice prepares two bits via the formula $c_1 = H(\vec{x} \cdot \vec{\lambda}_1)$ and $c_2 = H(\vec{x} \cdot \vec{\lambda}_2)$ and sends them to Bob.
3. Bob flips each vector $\vec{\lambda}_i$ when the corresponding bit c_i is zero. This is equivalent to set $\vec{\lambda}_i' := (-1)^{1+c_i} \vec{\lambda}_i$.
4. Instead of performing a POVM with elements $B_k = 2p_k |\vec{y}_k\rangle \langle \vec{y}_k|$, Bob picks one vector $\vec{y}_k$ from the set $\{\vec{y}_k\}$ according to the probabilities $\{p_k\}$. Then he sets $\vec{\lambda} := \vec{\lambda}_1'$ if $|\vec{\lambda}_1' \cdot \vec{y}_k| \geq |\vec{\lambda}_2' \cdot \vec{y}_k|$ and $\vec{\lambda} := \vec{\lambda}_2'$ otherwise. Finally, Bob outputs k with probability

$$p_B(k|\{B_k\}, \vec{\lambda}) = \frac{p_k \, \Theta(\vec{y}_k \cdot \vec{\lambda})}{\sum_j^N p_j \, \Theta(\vec{y}_j \cdot \vec{\lambda})}$$

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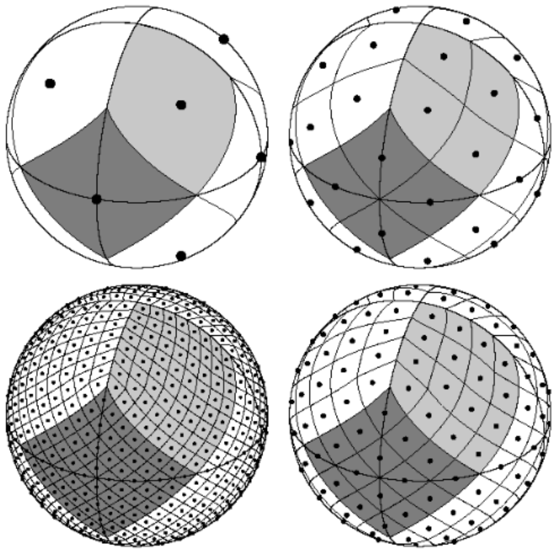
PREPARE-AND-MEASURE CLASSICAL PROTOCOL: STATE PREPARATION

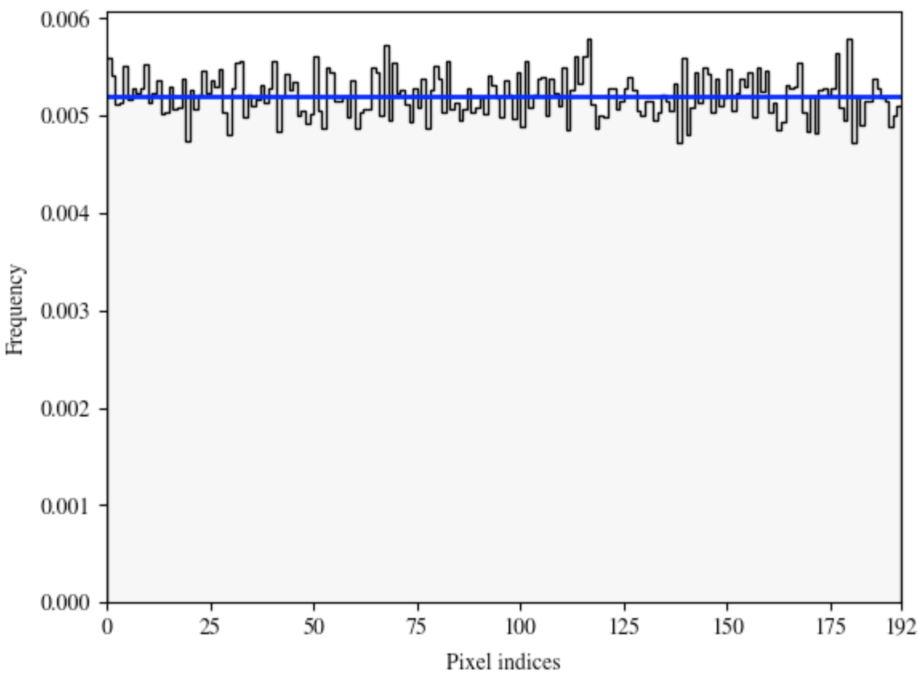
To produce a **random qubit pure state**, we should obtain a random unitary matrix and then apply the unitary transformation to the zero qubit state, resembling the time evolution of a qubit from a zero initial state.

The random unitary matrix can be generated by just building a matrix of normally distributed complex numbers, and then apply the **Gram-Schmidt QR decomposition to orthogonalize the matrix**.

HOW WE VALIDATE THE RANDOM QUBIT STATE DISTRIBUTION?

Hierarchical **E**qual **A**rea iso**L**atitude **Pix**elisation (**HEALPix**)
of the Bloch sphere





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PREPARE-AND-MEASURE CLASSICAL PROTOCOL: RANK-1 POVM GENERATION

Every **POVM** can be written as a coarse graining of rank 1 projectors, such that the protocol implementation can restrict without any loss in generality to POVMs proportional to rank-1 projectors.

As described by *Sentís et al.* the conditions under which a set of N arbitrary rank-1 operators $\{B_k\}$ comprises a qubit POVM, can be equivalently written in a system of **four linear equations.** The existence of the set $\{a_k\}$ has a direct translation into a linear programming feasibility problem we would have to solve **computationally**.

$$\sum_{k=1}^{N} a_k = 2$$

$$\sum_{k=1}^{N} a_k \vec{y}_k = \vec{0}$$

As an example, to build a random POVM set of $N = 4$ elements, we could apply the following procedure:

1. Assign two rank-1 operators as projective measurement elements $E_i = |v_i\rangle\langle v_i|$ with unknown weights $\{a_i\}$, where $i = 1, 2$.
2. Apply the closure relation such that the third rank-1 operator is $E_3 = \mathbb{1} - \sum_{i=1}^{2} E_i$. Note that this will not be necessarily a rank-1 operator.
3. Diagonalize E_3 to obtain the relevant qubit states as eigenvectors $|v_3\rangle$ and $|v_4\rangle$.
4. Convert all qubit states $|v_i\rangle$ to Bloch vectors $\vec{y}_i$, where $i = 1, 2, ...4$.
5. Solve the linear programming feasibility problem

$$\begin{array}{ll} \text{find} & x = \{a_1, a_2, \ldots, a_N\} \\ \text{subject to} & Ax = b \text{ where column } A_{*k} = (\vec{y}_k, 1), \text{ and } b = (\vec{0}, 2) \\ & x \geq 0 \end{array}$$

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PREPARE-AND-MEASURE CLASSICAL PROTOCOL: SIMULATIONS

Simulations run using **random states and POVM measures,** and also **well-known POVMs (e.g. Cross, Trine, SIC-POVM), all reproducing** the quantum **probabilities** with **extraordinary accuracy**.

Cross-POVM $\mathbb{P}_4 = \{\frac{1}{2}\left|0\right\rangle\left\langle 0\right|, \frac{1}{2}\left|1\right\rangle\left\langle 1\right|, \frac{1}{2}\left|+\right\rangle\left\langle +\right|, \frac{1}{2}\left|-\right\rangle\left\langle -\right|\}$

SIC-POVM

$\mathbb{P}_4 = \{E_1, E_2, E_3, E_4\}$ and $E_k = \frac{1}{2}\left|\Psi_k\right\rangle\left\langle\Psi_k\right|$, where

$$\left|\Psi_1\right\rangle = \left|0\right\rangle$$

$$\left|\Psi_2\right\rangle = \frac{1}{\sqrt{3}}\left|0\right\rangle + \sqrt{\frac{2}{3}}\left|1\right\rangle$$

$$\left|\Psi_3\right\rangle = \frac{1}{\sqrt{3}}\left|0\right\rangle + \sqrt{\frac{2}{3}}\, e^{i\frac{2\pi}{3}}\left|1\right\rangle$$

$$\left|\Psi_4\right\rangle = \frac{1}{\sqrt{3}}\left|0\right\rangle + \sqrt{\frac{2}{3}}\, e^{i\frac{4\pi}{3}}\left|1\right\rangle$$

Scenario	Probabilities			
Cross-POVM[1]	$0.3749^{0.3750}$	$0.1250^{0.1250}$	$0.0625^{0.0625}$	$0.4376^{0.4375}$
Trine-POVM[2]	$0.4998^{0.5000}$	$0.0335^{0.0335}$	$0.4667^{0.4665}$	-
SIC-POVM[3]	$0.3751^{0.3750}$	$0.0315^{0.0316}$	$0.3851^{0.3851}$	$0.2082^{0.2083}$
Random-PVM[4]	$0.9669^{0.9669}$	$0.0331^{0.0331}$	-	-
Random-POVM[5]	$0.0097^{0.0097}$	$0.0057^{0.0057}$	$0.8825^{0.8825}$	$0.1021^{0.1021}$
Random-POVM[6]	$0.2386^{0.2389}$	$0.1242^{0.1242}$	$0.6341^{0.6337}$	$0.0031^{0.0031}$

Trine-POVM

$\mathbb{P}_3 = \{E_1, E_2, E_3\}$ and $E_k = \frac{2}{3}\left|\Psi_k\right\rangle\left\langle\Psi_k\right|$, where

$$\left|\Psi_1\right\rangle = \left|0\right\rangle$$

$$\left|\Psi_2\right\rangle = \frac{1}{2}\left|0\right\rangle + \frac{\sqrt{3}}{2}\left|1\right\rangle$$

$$\left|\Psi_3\right\rangle = \frac{1}{2}\left|0\right\rangle - \frac{\sqrt{3}}{2}\left|1\right\rangle$$

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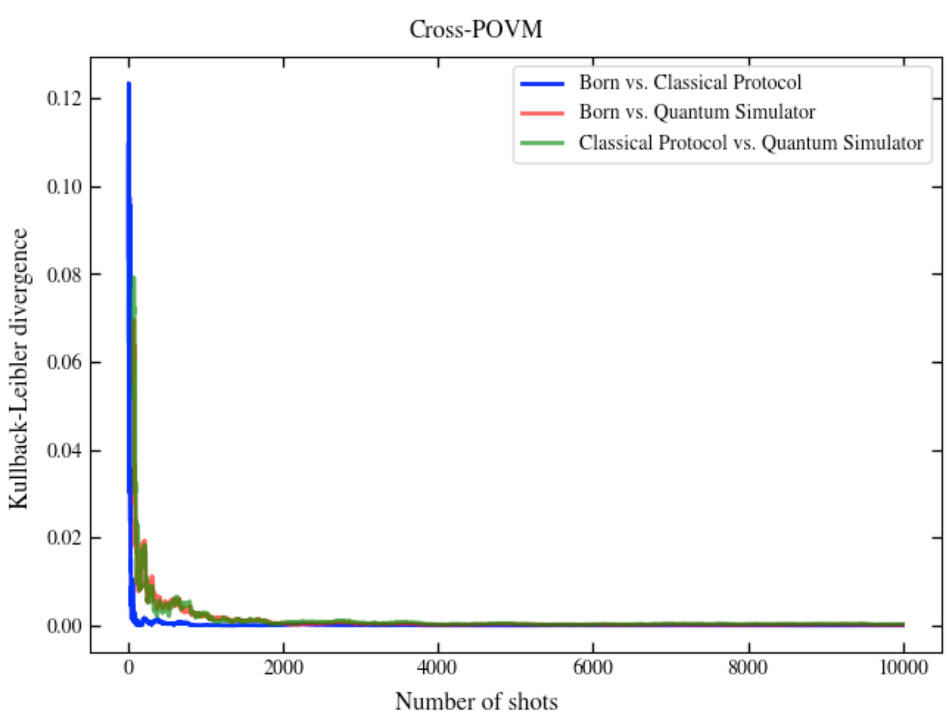
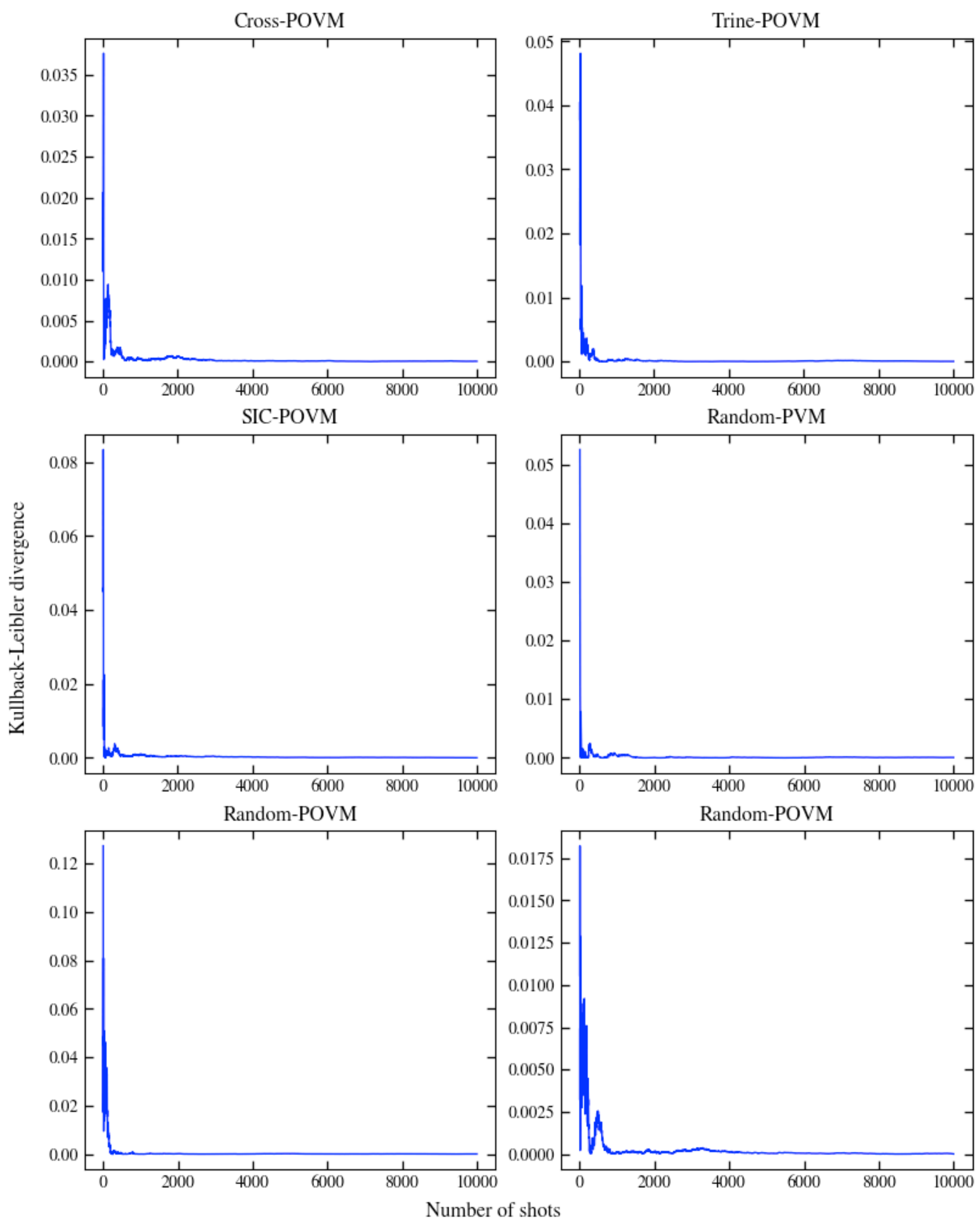
PREPARE-AND-MEASURE CLASSICAL PROTOCOL: SIMULATIONS

The **relative entropy distance**, or Kullback-Leibler divergence, among the theoretical and classical simulation probability distributions:

$$D_{KL}(P||Q) = \sum_{x \in \mathcal{X}} P(x)\ log \frac{P(x)}{Q(x)}$$





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BELL SCENARIO

Bipartite quantum system of two entangled and separated qubits:

ALICE | BOB

ALICE: Chooses a **random local measurement** A_x among two possible observables $\{A_0, A_1\}$

Output a_x

BOB: Chooses a **random local measurement** B_y among two possible observables $\{B_0, B_1\}$

Output b_y

$$p_{A_x,B_y}(a_x,b_y) = tr[|a_x\rangle\langle a_x| \otimes |b_y\rangle\langle b_y|\Phi_{ij}\rangle\langle\Phi_{ij}|]$$

- BELL CORRELATIONS

$$p_C(a_x,b_y|A_x,B_y) = \int_\lambda d\lambda\, \pi(\lambda) p_A(c|A_x,\lambda)\; p_B(b_y|B_y,c,\lambda)$$

Shared randomness (λ)

Classical information (c)

$$\forall A_x, B_y : \quad p_C(a_x,b_y|A_x,B_y) = p_{A_x,B_y}(a_x,b_y)$$

Success criteria

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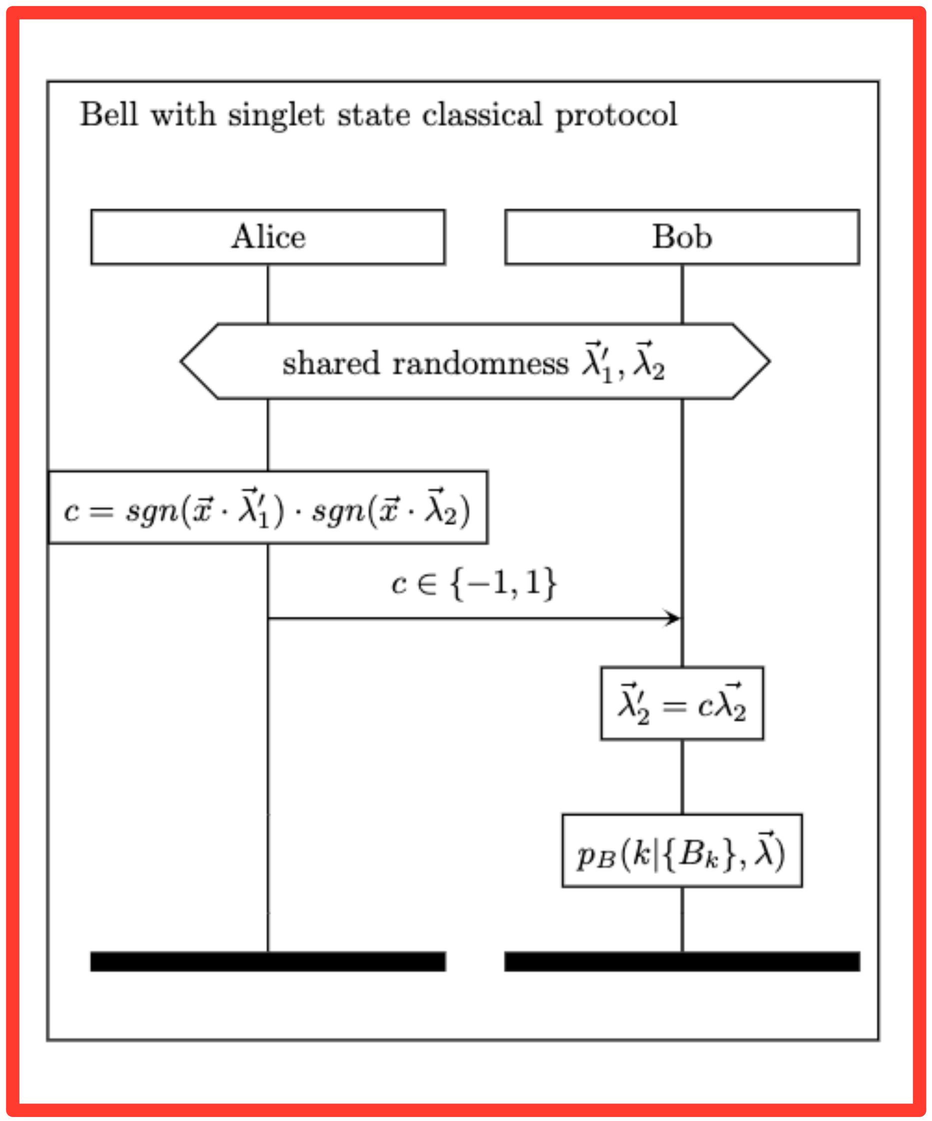
BELL CLASSICAL PROTOCOL



1. Alice and Bob share two normalized vectors $\vec{\lambda}'_1, \vec{\lambda}_2 \in \mathbb{R}^3$, which are uniformly and independently distributed on the unit radius sphere S_2.
2. Instead of performing a measurement with projectors $|\pm\vec{x}\rangle\langle\pm\vec{x}| = (\mathbb{1} + \vec{x} \cdot \vec{\sigma})/2$, Alice outputs $a = -sgn(\vec{x} \cdot \vec{\lambda}'_1)$ and sends the bit $c = sgn(\vec{x} \cdot \vec{\lambda}'_1) \cdot sgn(\vec{x} \cdot \vec{\lambda}_2)$ to Bob, where

$$sgn(z) = \begin{cases} 1 & \text{when } z \geq 0 \\ -1 & \text{when } z < 0 \end{cases} \tag{14}$$

3. Bob flips the vector $\vec{\lambda}_2$ if and only if $c = -1$, i.e. he sets $\vec{\lambda}'_2 := c\vec{\lambda}_2$.
4. Same as step 4 in the previous prepare-and-measure protocol.

$$p_B(k|\{B_k\}, \vec{\lambda}) = \frac{p_k \; \Theta(\vec{y}_k \cdot \vec{\lambda})}{\sum_j^N p_j \; \Theta(\vec{y}_j \cdot \vec{\lambda})}$$

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BELL CLASSICAL PROTOCOL: SIMULATIONS

Simulations run using **Bell singlet state** and **local projective measurements**, **reproducing joint probabilities** with **extraordinary accuracy**.

(a_x, b_y)	$p_C(a_x, b_y \vert A_x, B_y)$				CHSH
	(A_0, B_0)	(A_0, B_1)	(A_1, B_0)	(A_1, B_1)	
$(+1,+1)$	$0.4268^{0.4267}$	$0.4269^{0.4267}$	$0.4267^{0.4267}$	$0.0734^{0.0732}$	-
$(+1,-1)$	$0.0731^{0.0732}$	$0.0732^{0.0732}$	$0.0732^{0.0732}$	$0.4265^{0.4267}$	-
$(-1,+1)$	$0.0732^{0.0732}$	$0.0733^{0.0732}$	$0.0732^{0.0732}$	$0.4270^{0.4267}$	-
$(-1,-1)$	$0.4268^{0.4267}$	$0.4267^{0.4267}$	$0.4269^{0.4267}$	$0.0731^{0.0732}$	-
$\mathbb{E}[A_x, B_y]$	0.7072	0.7073	0.7072	-0.7070	2.8287

Table 5: Probability outcomes of a classical Bell simulation after 10^7 shots. The entanglement state is the Bell singlet state $|\Psi^-\rangle = (|00\rangle - |11\rangle)/\sqrt{2}$, and the observables chosen are $A_0 = Z$, $A_1 = X$, $B_0 = -(X+Z)/\sqrt{2}$, $B_1 = (X-Z)1/\sqrt{2}$. For the purpose of comparison, theoretical probabilities calculated using Born's rule are presented in superscript blue color.

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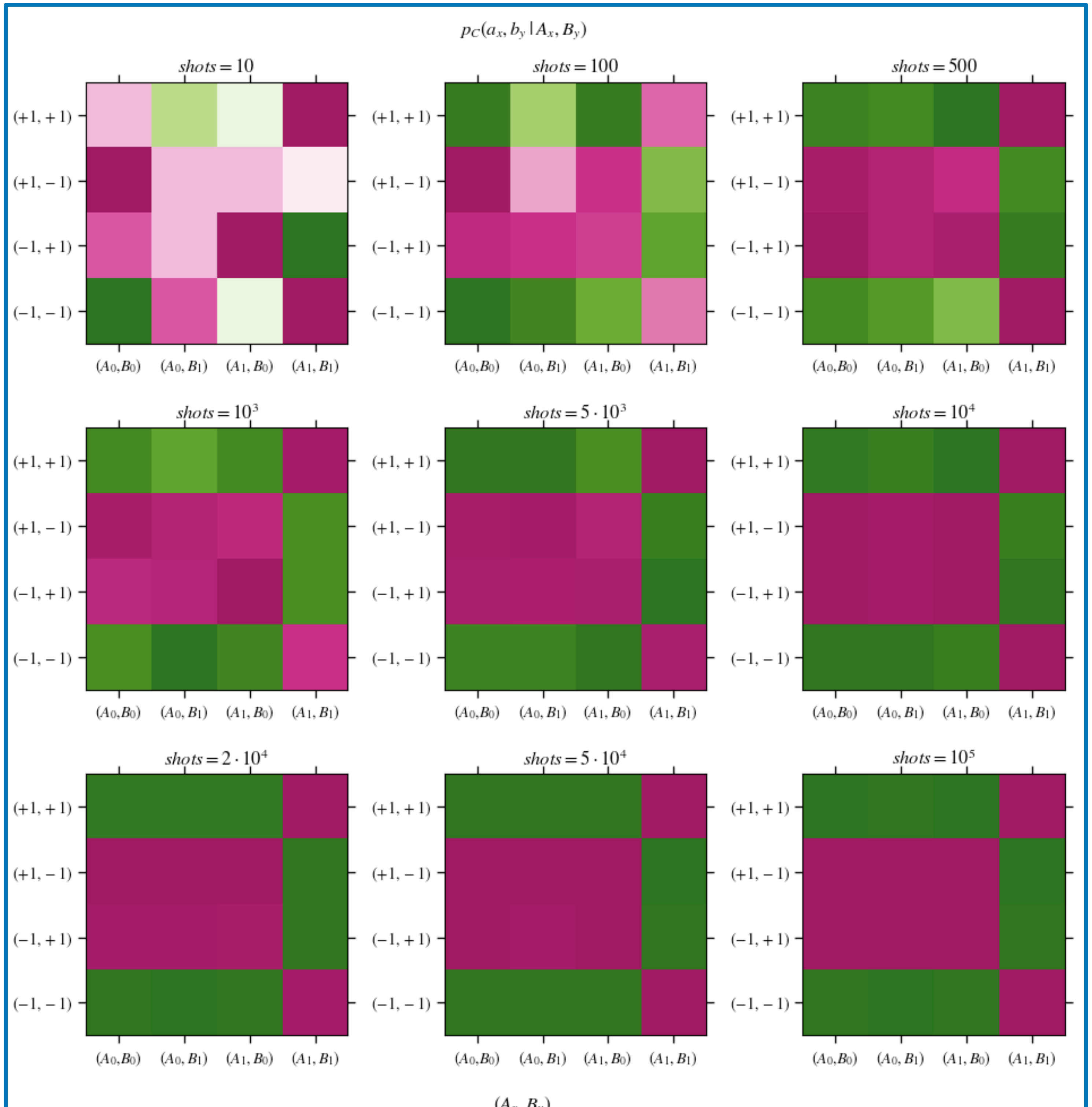

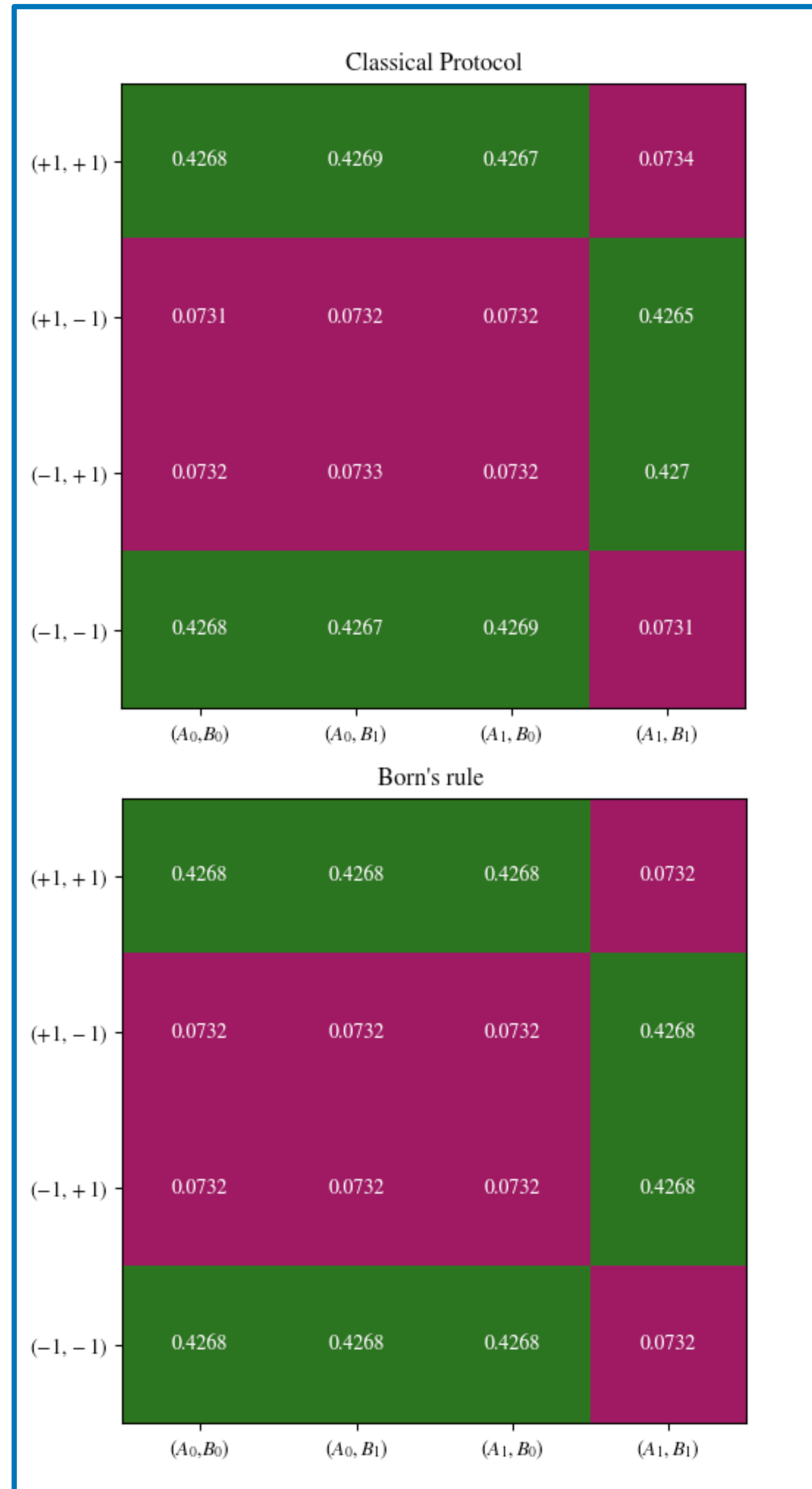

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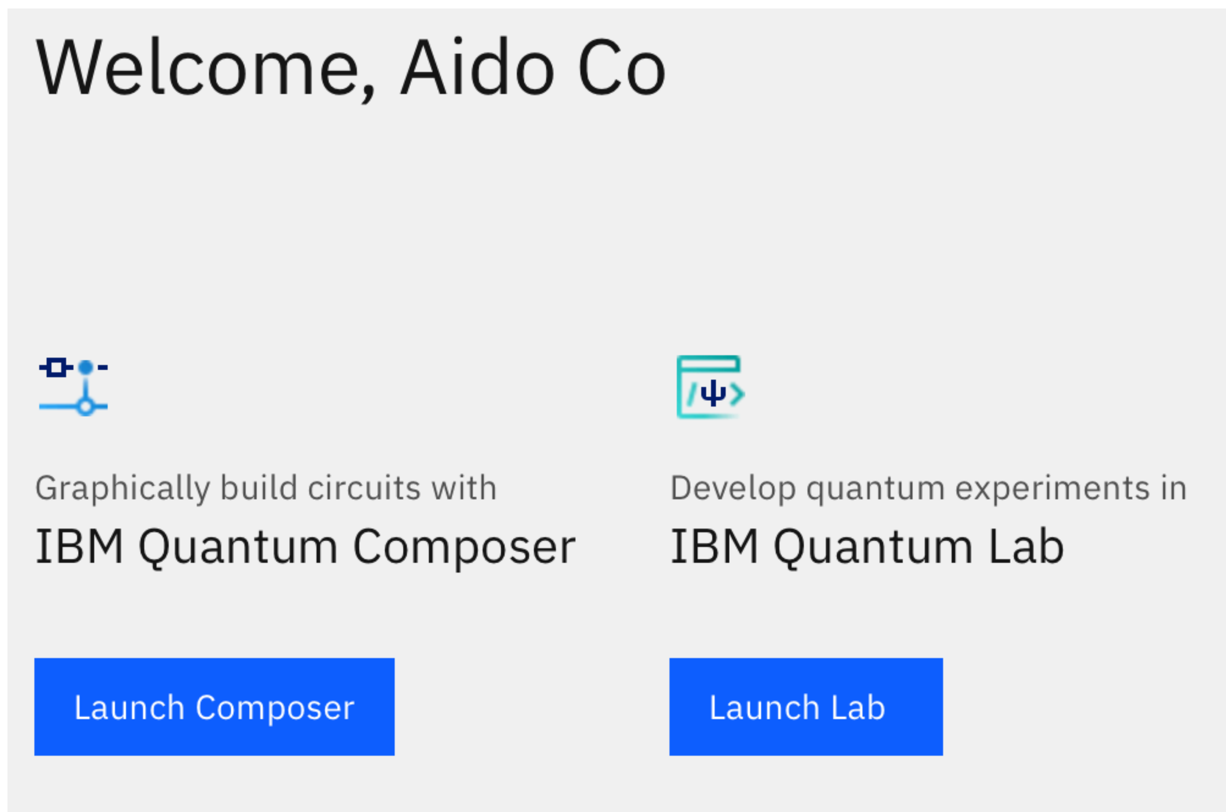
4. QUANTUM SIMULATION WITH QUANTUM COMPUTERS

We worked with QISKIT (IBM): https://quantum-computing.ibm.com



2 PROGRAMING PATHS: CIRCUIT OR CODE

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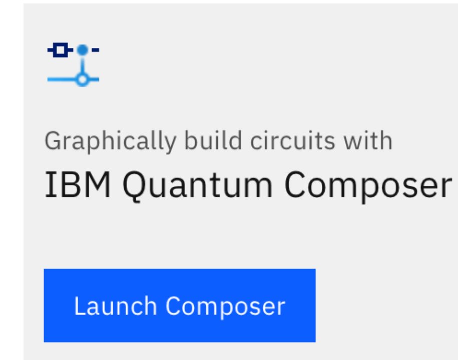

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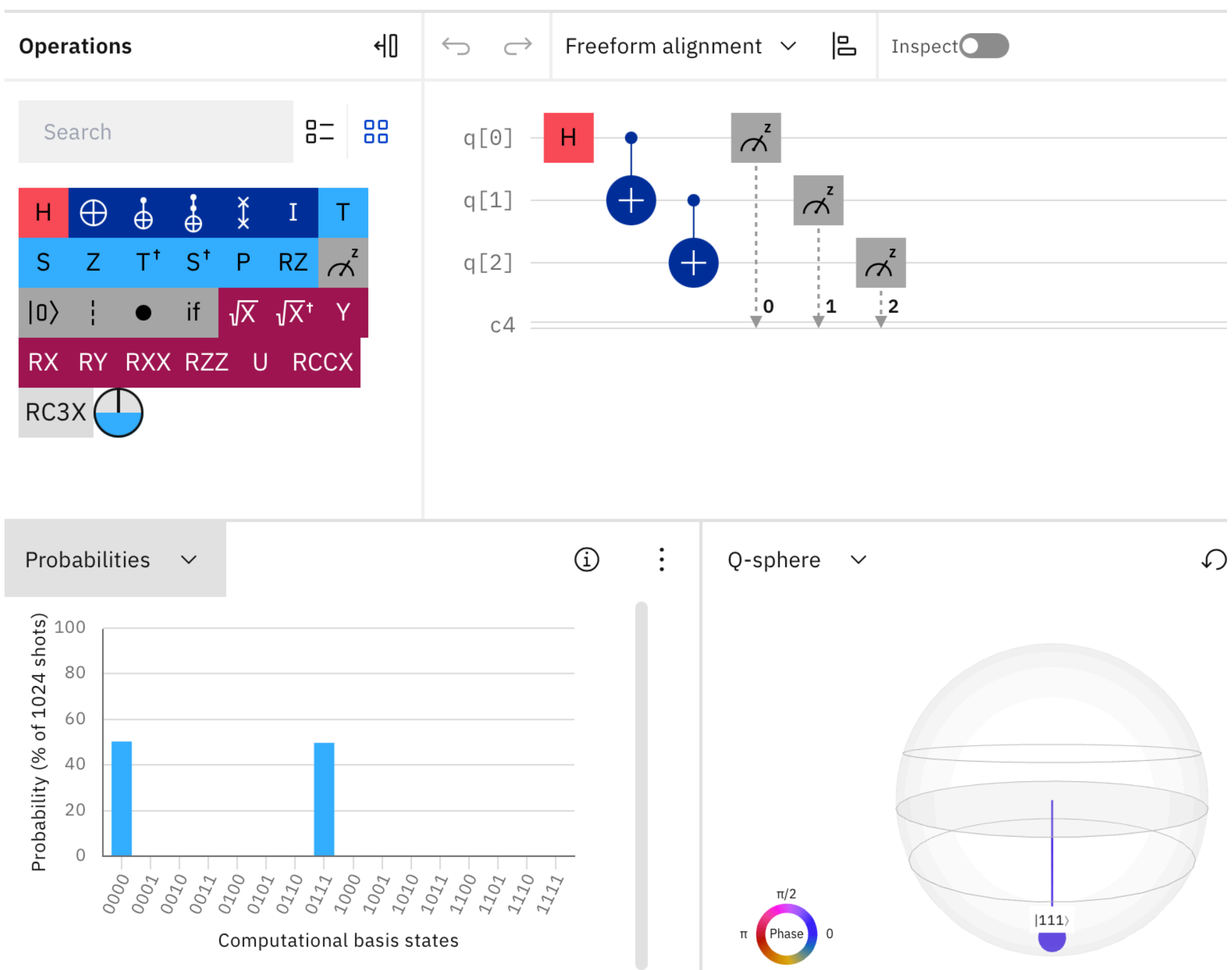
Circuit example in IBM Quantum Composer (not project code):



- EASY TO USE.
- SIMPLE.
- VISUAL.
- EASY SIMULATION IN LOCAL COMPUTER.
- EASY EXECUTION IN QC.

- LIMITED USES: IN OUR CASE CIRCUIT IS NOT PREPARED TO CREATE A UNITARY MATRIX OR POVM MEASUREMENTS.

- ALL COMPLEX QUANTUM PROGRAMS DEVELOPED BY CODE.

4. QUANTUM SIMULATION WITH QUANTUM COMPUTERS

Project program in IBM Quantum Lab:

Develop quantum experiments in
IBM Quantum Lab

Launch Lab

QS-10000 SHOTS- 01042

Code

Python 3 (ipykernel)

```
[1]: from qiskit import transpile, assemble
     from qiskit import execute, Aer, IBMQ, QuantumRegister, ClassicalRegister
     from qiskit.visualization import plot_histogram
     from qiskit import QuantumCircuit
     from qiskit.extensions import UnitaryGate
     from qiskit.providers.aer import QasmSimulator
     from qiskit.tools.jupyter import *
     from qiskit.visualization import *
     from ibm_quantum_widgets import *
     import math
     simulator = QasmSimulator()

[2]: qc = QuantumCircuit(2,2)

[3]: psi=((3+1.j*math.sqrt(3))/4.,-0.5)

[4]: U =  [[ 0.70710678+0.j, 0.+0.j, 0.70710678+0.j, 0.+0.j],
      [ 0.+0.j, 0.70710678+0.j, 0.  +0.j, 0.70710678+0.j],
      [ 0.5-0.j, 0.5+0.j, -0.5+0.j, -0.5+0.j],
      [-0.5+0.j, 0.5+0.j, 0.5+0.j, -0.5+0.j]]

[5]: qc.initialize (psi,0)
     qc.unitary(U,[0,1])
     qc.measure([0,1],[0,1])
     qc.draw()
```

- PYTHON: high level programming language.
- With the relevant packages the program can be **simulated locally**, or even **launched** on IBM **quantum computers.**
- Programming option used in the project.

Notebook

Python 3 (ipykernel)

To simulate the quantum program locally we use
AER SIMULATION.

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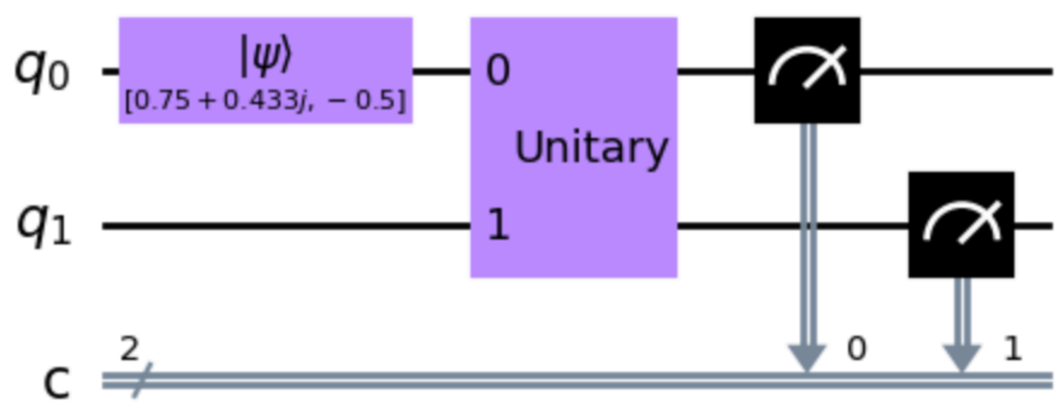
Project program in IBM Quantum Lab (**QUANTUM SIMULATION**):



Above: Quantum program drawing

- In order to compare the quantum simulation program with the classical program. We must fix a state and unitary matrix with characterized POVM (Naimark extension).

```
psi=((3+1.j*math.sqrt(3))/4.,-0.5)
```

```
U =  [[ 0.70710678+0.j, 0.+0.j, 0.70710678+0.j, 0.+0.j],
 [ 0.+0.j, 0.70710678+0.j, 0. +0.j, 0.70710678+0.j],
 [- 0.5+0.j, -0.5+0.j, 0.5+0.j, 0.5+0.j],
 [-0.5+0.j, 0.5+0.j, 0.5+0.j, -0.5+0.j]]
```

- QISKIT libraries are not prepared to POVMs, the only measurements allow are PVMs.

$$|\Psi\rangle = \frac{3+i\sqrt{3}}{4}|0\rangle - \frac{1}{2}|1\rangle$$

$$\mathbb{P}_4 = \{\frac{1}{2}|0\rangle\langle 0|, \frac{1}{2}|1\rangle\langle 1|, \frac{1}{2}|+\rangle\langle +|, \frac{1}{2}|-\rangle\langle -|\}$$

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4. QUANTUM SIMULATION WITH QUANTUM COMPUTERS

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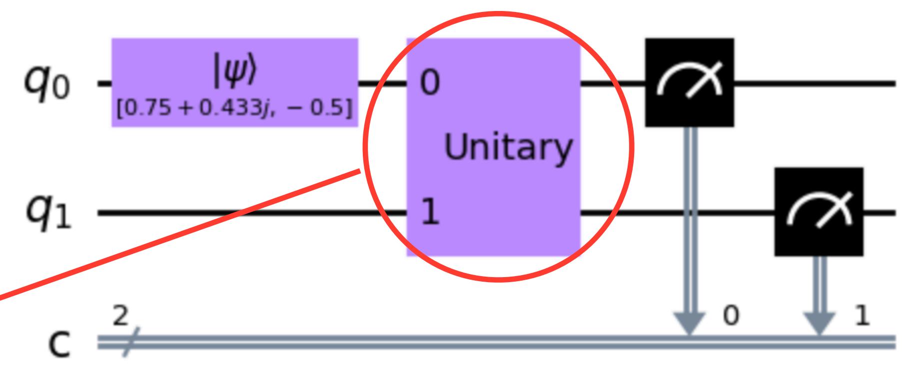
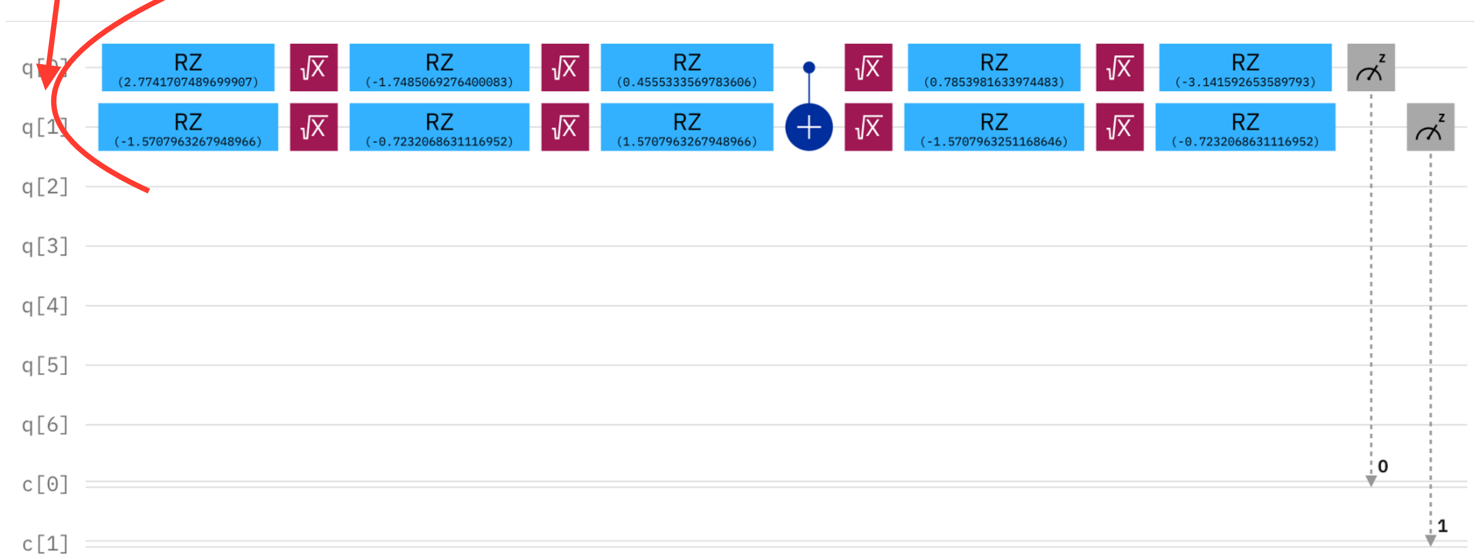
Project program in IBM Quantum Lab (**QUANTUM SIMULATION**):



- To decompose the unitary matrix in standard gates, we have the theoretical option in the book Quantum Information by Nielsen and Chuang, or Qiskit Transpile (Quantum process rewriting).

```
backend = Aer.get_backend('aer_simulator')
qc_transpiled = transpile (qc, backend)
qc_transpiled.draw()
```

To simulate the quantum program locally we use
AER SIMULATION.



Above: Quantum circuit after transpile

- Transpile is not efficient at 100%. For descomposing a toffoli gate uses 16cnot when theoretically the minimum would be 6.
- But for our program is good enough. One CNOT in transpile and in Nielsen and Chuang unitary decomposition.

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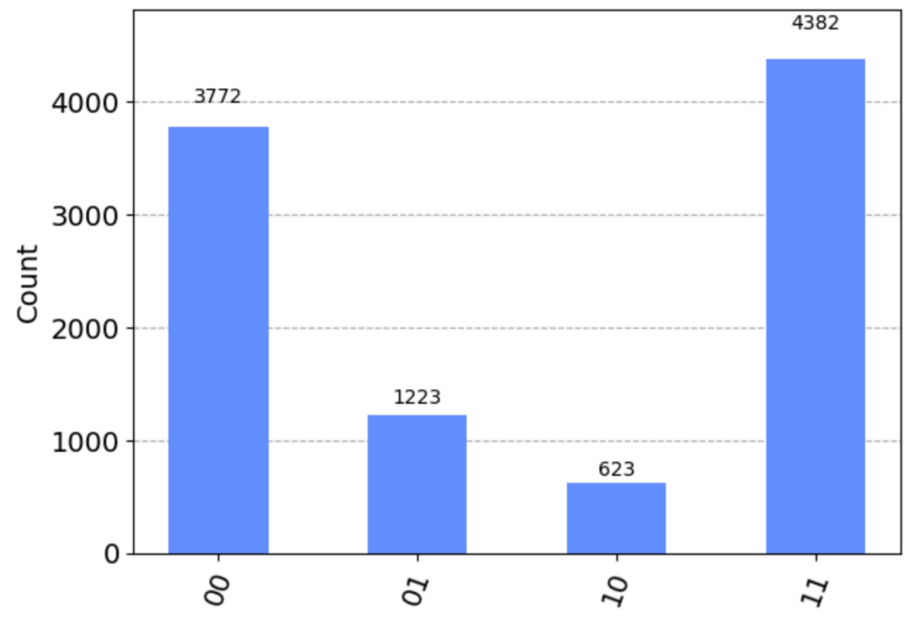
Project program in IBM Quantum Lab (**QUANTUM SIMULATION**):

SAME RESULTS IN QUANTUM AND CLASSICAL PROTOCOL:

LOVE IS IN THE “AER”



NEXT STEP: EXECUTE QUANTUM PROGRAM IN QC

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Project program in IBM Quantum Lab (QUANTUM COMPUTER):

Recent jobs View all

0 Pending | 68 Completed

- Completed: Apr 21, 2023 4:03 PM
- Completed: Apr 21, 2023 3:59 PM
- Completed: Apr 18, 2023 7:01 PM
- Completed: Apr 18, 2023 6:17 PM
- Completed: Apr 15, 2023 12:39 PM
- Completed: Apr 15, 2023 11:37 AM

- For understanding Real Quantum Computation, tools and error, we run 68 programs in 5 Quantum Computers.

Name	Qubits ↓	QV	CLOPS	Status	Total pending jobs
ibm_perth	7	32	2.9K	Online	203
ibm_lagos	7	32	2.7K	Online	82
ibm_nairobi	7	32	2.6K	Online	71
ibm_oslo	7	32	2.6K	Online	9
ibmq_jakarta	7	16	2.4K	Online	209
ibmq_manila	5	32	2.8K	Online	119
ibmq_quito	5	16	2.5K	Online	1
ibmq_belem	5	16	2.5K	Online	3
ibmq_lima	5	8	2.7K	Online	23

- We choose the 5 QC (free available) with best relation QV, CLOPS, median CNOT error and median Readout error.

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Project program in IBM Quantum Lab (**QUANTUM COMPUTER**):

```
IBMQ.load_account()
provider = IBMQ.get_provider(hub = 'ibm-q',group = 'open', project = 'main' )
qcomp = provider.get_backend ('ibm_perth')
```

```
qc_transpiled = transpile (qc, backend=qcomp)
job = execute (qc_transpiled, backend=qcomp,shots=1000)
job_monitor(job)
result = job.result()
plot_histogram(result.get_counts(qc_transpiled))
```

```
Job Status: job is queued (128)
```

Above: Quantum program execution in QC perth with 1000 shots

	QUBITS	QV	CIRCUIT LAYER	Median CNOT ERROR	Median ReadOut Error
NAIROBI	7	32	2.6K	0,01357	0,0227
PERTH	7	32	2.9K	0,01733	0,0188
OSLO	7	32	2.6K	0,01	0,01667
JAKARTA	7	16	2.4K	0,00773	0,0258
LAGOS	7	32	2.7K	0,007243	0,0145

- In theory LAGOS is the best quantum computer, let's check the results.

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Quantum compilation Results:

Why we run program in 5 QC?

Qiskit works automatically with 4000 shots.

Initially, We launched two QC programs in Nairobi, with 4000 shots and 10000 shots,

Contrary to what theory told us, the **more shots** we implement, the **worse** results we got ….

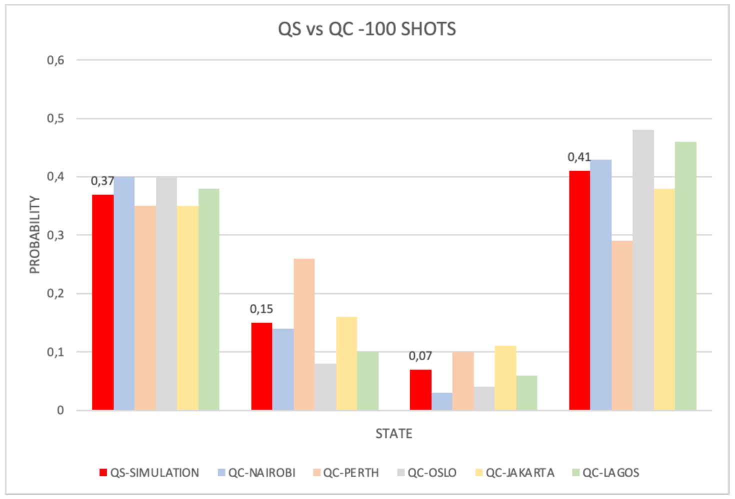
It was not logical at all, that's why:

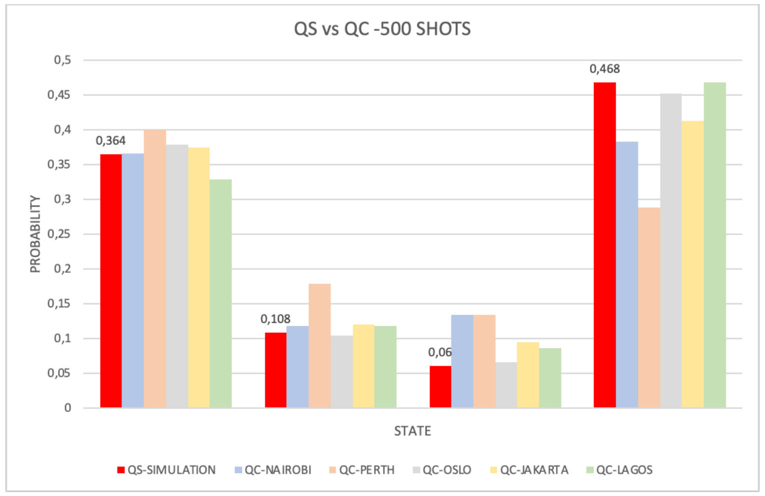
We studied the behavior of 5QC with the same number of Qubits, for different number of shots: 100, 500, 1.000, 4.000, 10.000 and 20.000 (20.000 is the maximum that can be launched in a QC for free).

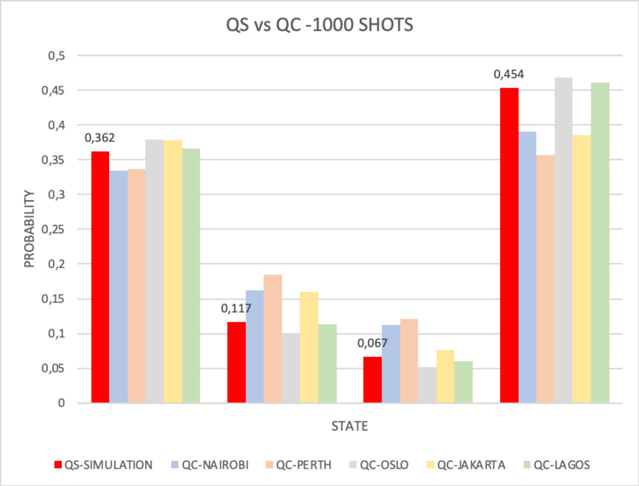
4. QUANTUM SIMULATION WITH QUANTUM COMPUTERS

Quantum compilation Results (Quantum Simulation vs Quantum Computation):

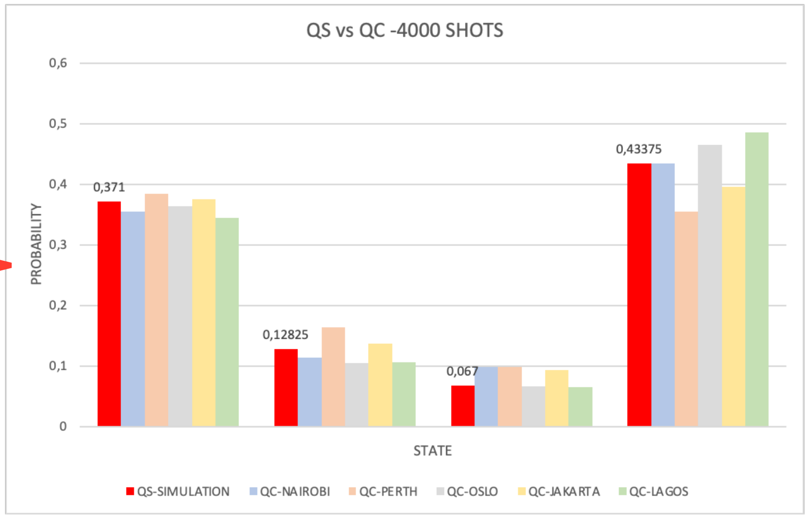
	100 SHOTS				500 SHOTS				1000 SHOTS				4000 SHOTS				10000 SHOTS				20000 SHOTS			
RESULTS	'00'	'01'	'10'	'11'	'00'	'01'	'10'	'11'	'00'	'01'	'10'	'11'	'00'	'01'	'10'	'11'	'00'	'01'	'10'	'11'	'00'	'01'	'10'	'11'
QS-SIMULATION	0,37	0,15	0,07	0,41	0,364	0,108	0,06	0,468	0,362	0,117	0,067	0,454	0,371	0,128	0,067	0,43375	0,377	0,122	0,062	0,438	0,38	0,12	0,06	0,44
QC-NAIROBI	0,4	0,14	0,03	0,43	0,366	0,118	0,134	0,382	0,335	0,162	0,112	0,391	0,354	0,113	0,098	0,4345	0,378	0,172	0,09	0,359	0,35	0,11	0,1	0,44
QC-PERTH	0,35	0,26	0,1	0,29	0,4	0,178	0,134	0,288	0,337	0,185	0,121	0,357	0,384	0,164	0,098	0,35475	0,37	0,187	0,118	0,325	0,33	0,15	0,11	0,41
QC-OSLO	0,4	0,08	0,04	0,48	0,378	0,104	0,066	0,452	0,379	0,1	0,052	0,469	0,364	0,105	0,067	0,46525	0,358	0,108	0,07	0,463	0,38	0,11	0,06	0,44
QC-JAKARTA	0,35	0,16	0,11	0,38	0,374	0,12	0,094	0,412	0,378	0,16	0,076	0,386	0,375	0,137	0,093	0,396	0,383	0,143	0,088	0,386	0,37	0,15	0,1	0,39
QC-LAGOS	0,38	0,1	0,06	0,46	0,328	0,118	0,086	0,468	0,366	0,113	0,06	0,461	0,345	0,106	0,065	0,485	0,357	0,113	0,063	0,467	0,37	0,13	0,07	0,43

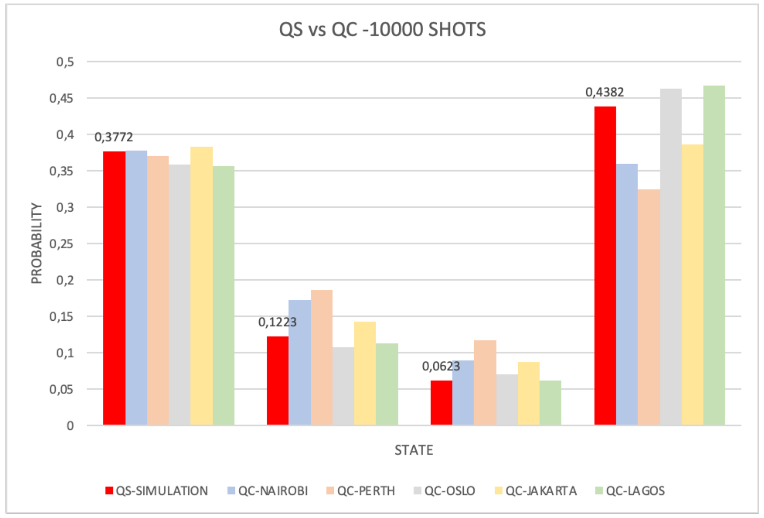


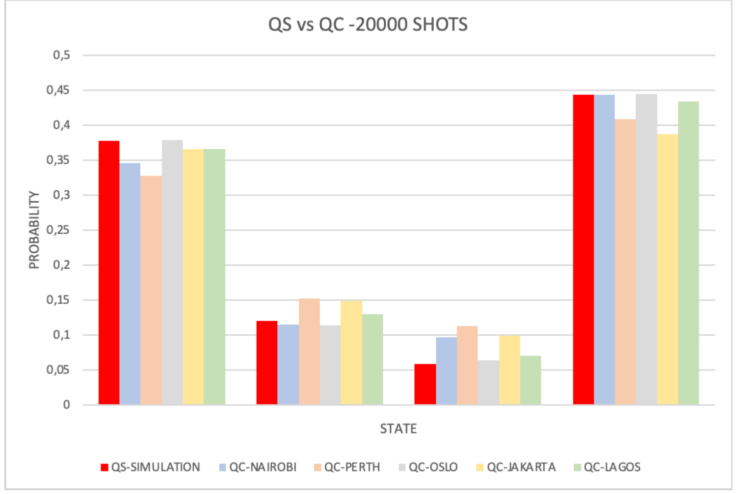




- We can appreciate the GAP between QS and QC
-







- We can NOT obtain any conclusion.

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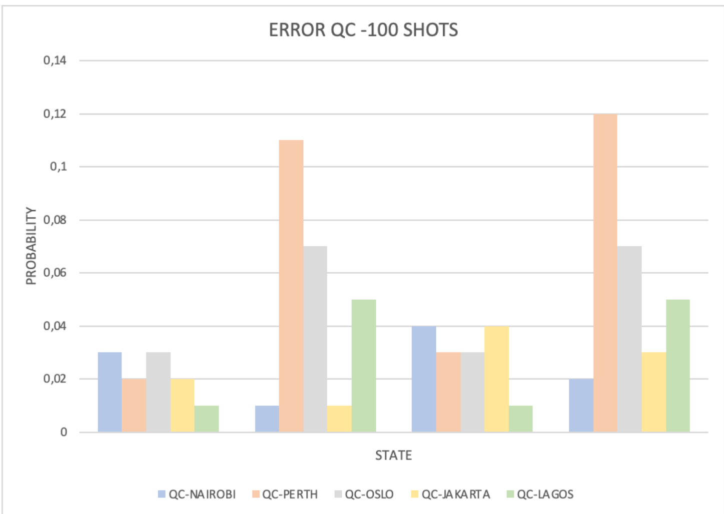
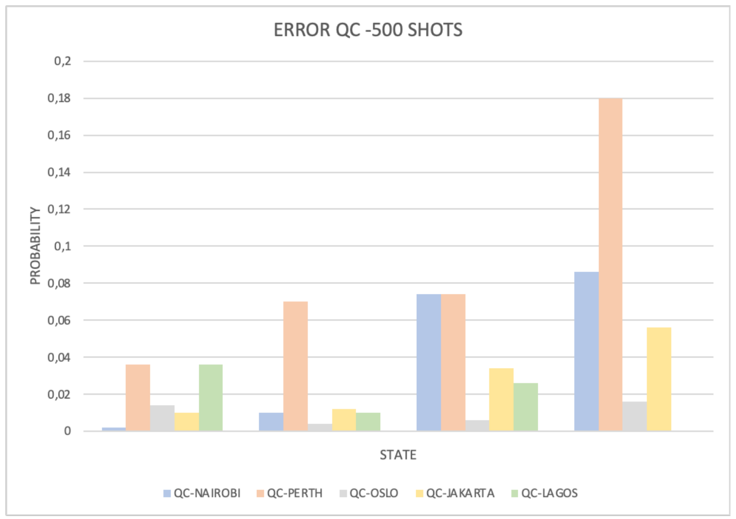
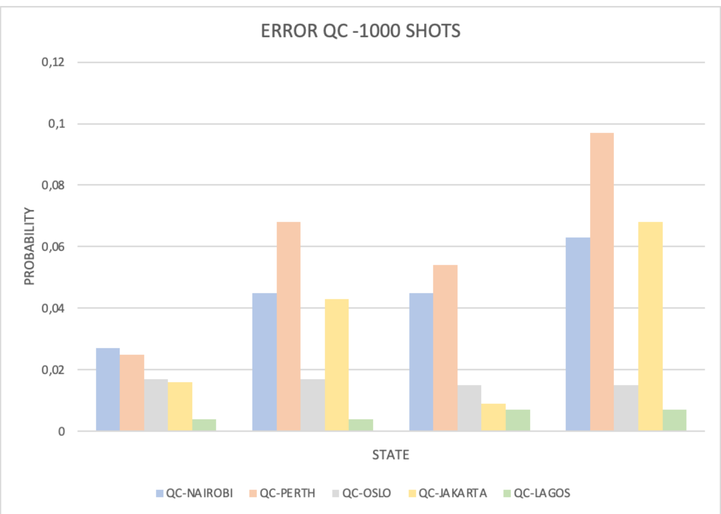
Quantum compilation Results (Error comparison):

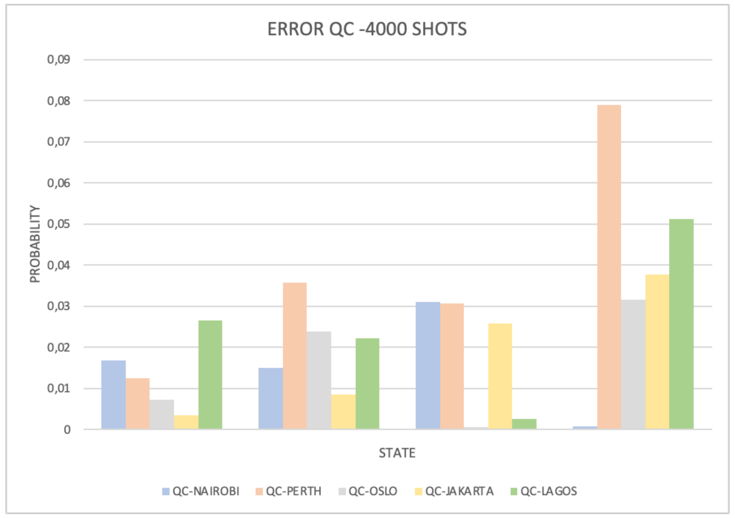
	100 SHOTS				500 SHOTS				1000 SHOTS				4000 SHOTS				10000 SHOTS				20000 SHOTS			
RESULTS	'00'	'01'	'10'	'11'	'00'	'01'	'10'	'11'	'00'	'01'	'10'	'11'	'00'	'01'	'10'	'11'	'00'	'01'	'10'	'11'	'00'	'01'	'10'	'11'
QC-NAIROBI	0,03	0,01	0,04	0,02	0,002	0,01	0,074	0,086	0,027	0,045	0,045	0,063	0,017	0,015	0,031	0,00075	0,001	0,05	0,028	0,079	0,03	0,01	0,04	0
QC-PERTH	0,02	0,11	0,03	0,12	0,036	0,07	0,074	0,18	0,025	0,068	0,054	0,097	0,013	0,036	0,031	0,079	0,007	0,064	0,055	0,113	0,05	0,03	0,05	0,04
QC-OSLO	0,03	0,07	0,03	0,07	0,014	0,004	0,006	0,016	0,017	0,017	0,015	0,015	0,007	0,024	5E-04	0,0315	0,019	0,014	0,008	0,025	0,000	0,007	0,006	0,001
QC-JAKARTA	0,02	0,01	0,04	0,03	0,01	0,012	0,034	0,056	0,016	0,043	0,009	0,068	0,004	0,009	0,026	0,03775	0,006	0,02	0,026	0,052	0,01	0,03	0,04	0,06
QC-LAGOS	0,01	0,05	0,01	0,05	0,036	0,01	0,026	0	0,004	0,004	0,007	0,007	0,027	0,022	0,003	0,05125	0,02	0,009	2E-04	0,029	0,01	0,01	0,01	0,01

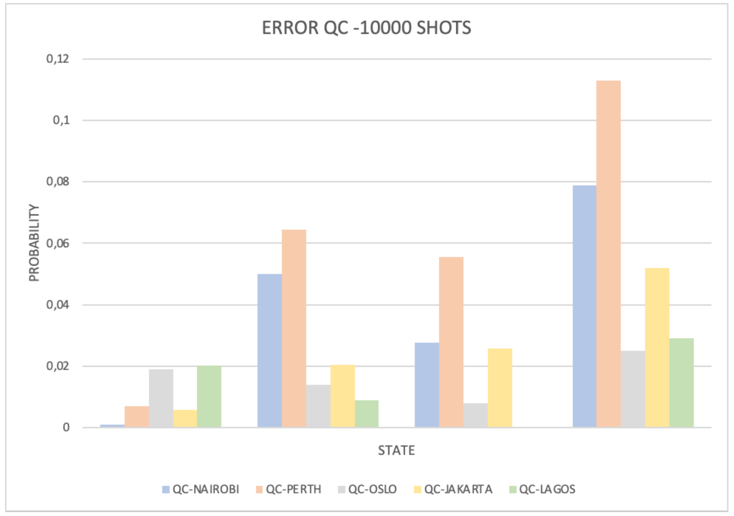


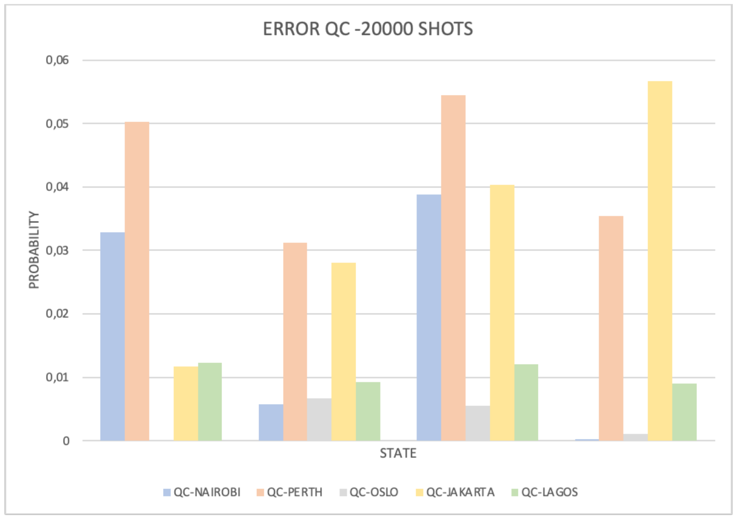




- We can easily see that PERTH, has more error than the other QC options.





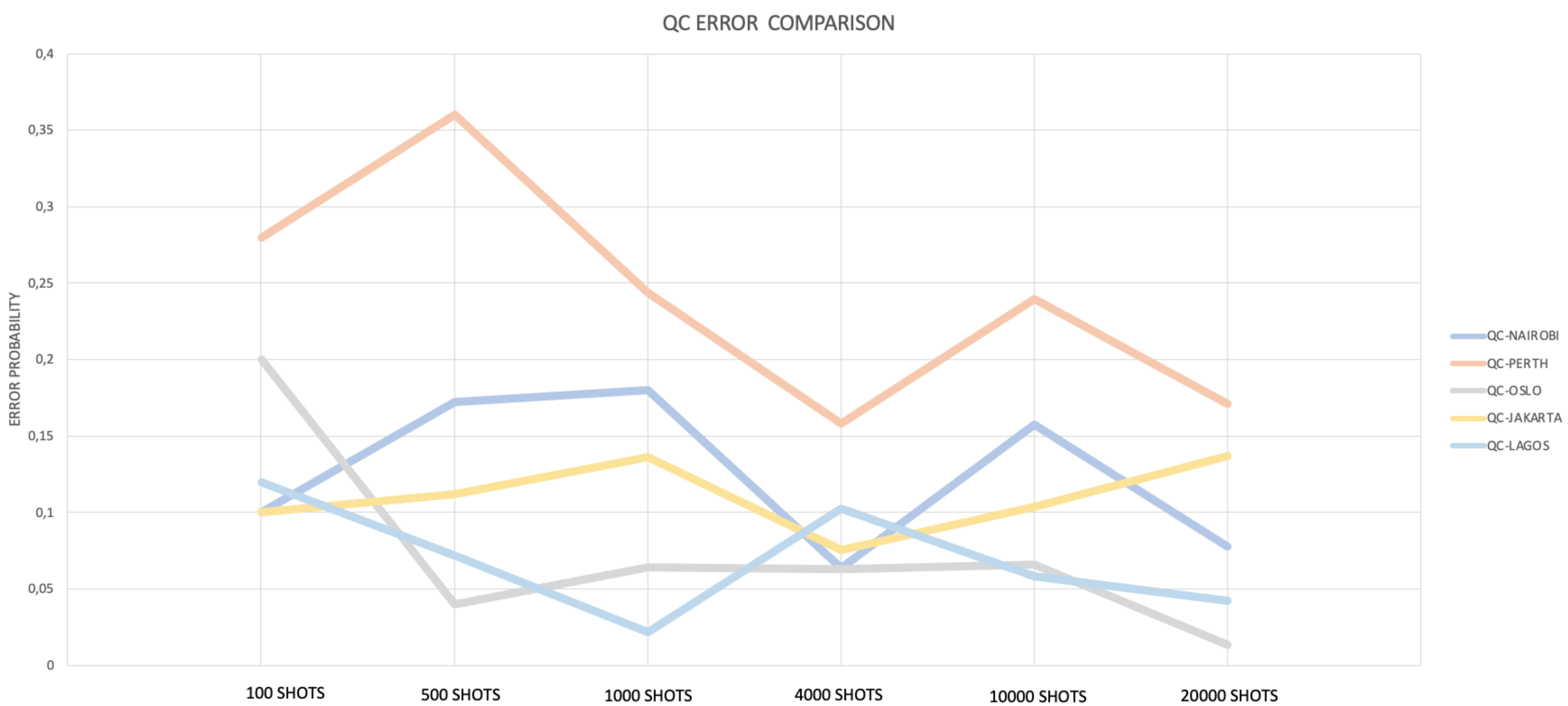


- We still can NOT obtain any conclusion.

4. QUANTUM SIMULATION WITH QUANTUM COMPUTERS

Quantum compilation Results (**Error comparison**):

	TOTAL ERROR					
RESULTS	**100 SHOTS**	**500 SHOTS**	**1000 SHOTS**	**4000 SHOTS**	**10000 SHOTS**	**20000 SHOTS**
QC-NAIROBI	0,1	0,172	0,18	0,0635	0,1576	0,0776
QC-PERTH	0,28	0,36	0,244	0,158	0,2396	0,1713
QC-OSLO	0,2	0,04	0,064	0,063	0,0658	0,0134
QC-JAKARTA	0,1	0,112	0,136	0,0755	0,1036	0,1368
QC-LAGOS	0,12	0,072	0,022	0,1025	0,0584	0,0426



4. QUANTUM SIMULATION WITH QUANTUM COMPUTERS

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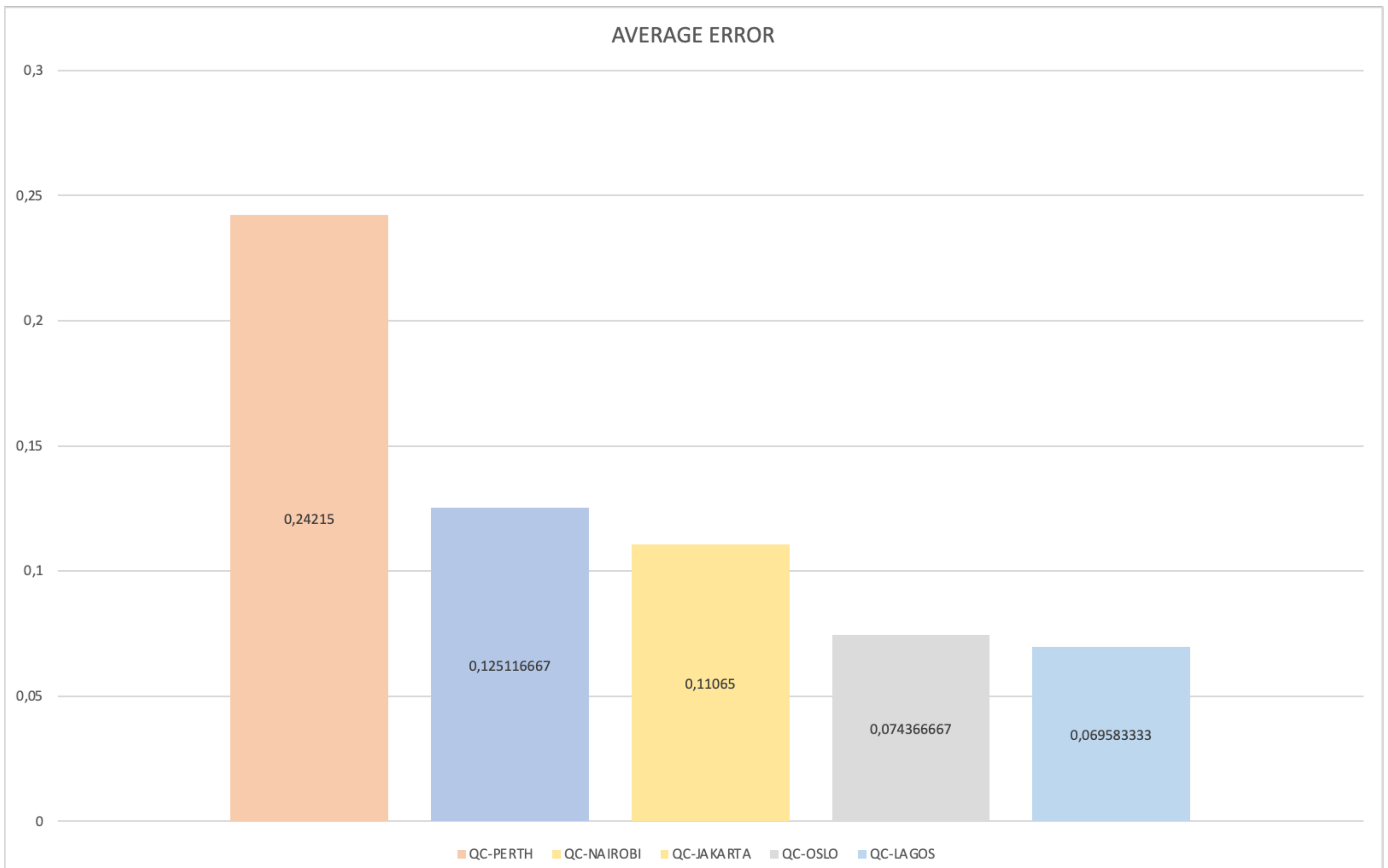
Quantum compilation Results (**Best Option for prepare and measure scenario**):



- We confirm that lagos is the best option, followed closely by Oslo.

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Quantum compilation Results:

1) Quantum computer statistics are different from Quantum Simulation, it is noted that the less noise the QC has, the **closer** statistics are to Quantum Simulation.

2) **All** five QC have better performance from **1.000** shots.

3) **Lagos** is the best QC option for prepare and measurement scenario, is the QC with lowest median CNOT error and median Readout error.

4) **Perth** is the worst QC option for prepare and measurement scenario by far.

5) **Oslo** has the closest approximation to Quantum Simulation (20.000 shots), for further work it will be interesting increasing the number of shots and check if it matches completely to simulation.

6) **Jakarta**, with worst QUANTUM VOLUME values, has given optimal results.

7) Running with 20.000 shots, **4/5** worked with less noise than in previous programs. Those who have given these values have the highest QUANTUM VOLUME (32).

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CONCLUSIONS:

- We have **executed and confirmed** the work of Martin J. Renner, Armin Tavakoli, Marco Túlio Quintino, 2022 , The classical cost of transmitting a qubit: two bits of communication are enough to classicaly simulate a qubit in a prepare-and-measure scenario.
- We also **executed and confirmed** the work of B.F. Toner and D.Bacon, 2004, Communication Cost of Simulating Bell Scenarios.
- We **simulated and executed** the prepare-and-measure protocol (POVM) in real quantum computers.
- The **running computing** time has been the same for the classical and quantum prepare-and-measure protocol.
- We **have obtained** very concrete data on the quantum computers performance: most relevant conclusion is: noise is actually being reduced in quantum computing by IBM.
- We can **easily identify** quantum computers **accuracy** by the theoretical values of each Quantum Computer, we also confirm statistically that the manufacturer values links to the results in a correct way.

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FURTHER WORK:

- Study classical communication applied to higher dimensional scenarios (**QUTRIT**), giving special attention to the new block sphere shape.

- Simulate and execute **Bell scenarios** protocol in quantum computers.

- Implement again the quantum protocol when the **transpile** function is more accurate (It depends on IBM Software improvements).

- Execute the quantum protocol over **20.000 shots** in the future.

- It would be really interesting **execute** this protocols in quantum computers from other manufactures (Qilimanjaro, Cirq, Dwave…), conclusions can be reached as to which technology is closest to theoretical simulation.

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WHAT WE HAVE LEARNED AS STUDENTS?

- **Lose the fear** with highly theoretical papers: We studied and understood more than **five papers** and **two books** in order to achieve the goals.

- **Perform** a Study Case, covering different quantum theoretical topics: **PREPARE- AND-MEASURE SCENARIOS with PVMs, POVMs and BELL SCENARIOS.**
  When we achieved a goal, we always had the feeling that we were starting at point zero, in order to learn and solve the new scenario.

- Implementation and simulation with a "**classic**" programming language (python)

- Implementation and simulation in **QISKIT**.

- Execute programs in **real** Quantum Computers.

- Hours and Hours of **learning**,
  Hours and Hours if **talks** with colleagues and professors,
  Hours and hours of **rethinking everything**, starting over and moving forward.
  Hours and hours of fun !!!!

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THANK YOU FOR YOUR ATTENTION

QUESTIONS….